A Paleo-Superstorm in Lake Athabasca, Canada?

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August 18th, 2005
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1.0 Abstract

A geologic survey of the William River Delta in Lake Athabasca, Canada, has shown GPR profiles which suggest the occurrence of millennial frequency storms. Simulations based on the STWAVE computational model were run. A wind speed of 50 m/s was selected with a direction of 45° (due southeast). The direction was chosen both because it yields the greatest size of the zone of breaking, as compared to other directions, as well as being the direction from which wind prevails near the William River Delta. The STWAVE simulation returned important wave output characteristics, such as average wave height and zone of breaking values. These output datasets were then put into two computational models, the mass balance model and the one-line shoreline erosion model. The results of the mass balance model suggested a storm duration of 5.61 days, while the more generalized results from the one-line model suggested a storm duration of 6 days. The result of each simulation neither proves nor excludes the possibility of a millennial storm event over Lake Athabasca.

2.0 Introduction

Lake Athabasca, a lake in Northern Canada, under extensive study by Dr. Derald Smith and his team of geologists from the University of Calgary, has shown some remarkable geologic formations. Though the lake itself is 7850 km^2 (Smith, Derald 6), most of the extreme geologic formations occur in a minuscule part of that overall area, near the delta of the north-flowing William River.

In the vicinity of the William River Delta, several structural sediment features are present. The smallest, j-shaped lensoid structures are created by small scale storms. Larger than these are, “ravinement surfaces that truncate the lensoid structures … [and] … are interpreted as having been eroded by rare storms of perhaps once in a 1000 years (millennial storms)” (Smith, Derald 21).

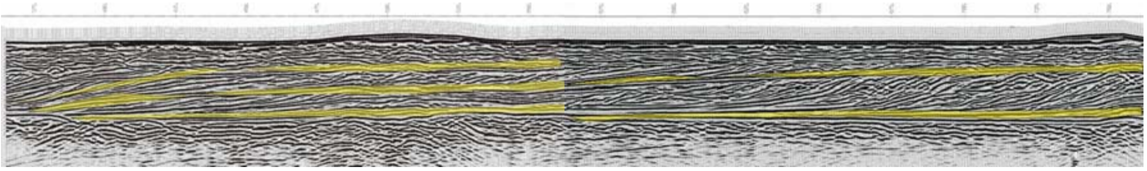


Figure 2.1: Lake Athabasca Ravinement Surface (Smith, Derald 61)

This idea of a “paleo-superstorm” is due to the fact that there are, “four of these ravinement surfaces in the strata between an approximate 4000 year OSL date (paleo-beach) and the present beach” (Smith, Derald 21). The general notion is that, “that millennial storm-generated waves and currents planed off the outer 200-400 m of the shelf zone,” and “at least some of the sediment was transported eastward by strong littoral currents” (Smith, Derald 21).

Evidence for the millennial occurrence is a geologic record which shows, “between 4000 and 2000 BP, we interpret three millennial-frequency storms. The data suggest that the frequency of severe storms is increasing with time. This trend may be attributed to periodic climatic cooling cycles with increased severity of storms in the last 5000 years, which culminated with the Little Ice Age between 650 and 150 BP.“

In order to assess whether or not this sediment profile could be cause by a storm system, a series of simulations were employed, using a number of different models and methodologies. First, however, input parameters must be derived from Lake Athabasca. A topographic lake bathymetry was provided by Dr. Smith. Another key fact gained from the Doctor was that, near the William River Delta, “the dominant and prevailing wind comes from the northwest” (Smith7).

Additionally, wind speed is also highly important for a storm event. Records from nearby Fort Chipewyan indicates, “maximum hourly velocities of 79 $^{km}/_{hr}$” (Smith, Derald 7) and the most severe event noted by the geologists had, “an average wind velocity of 63 $^{km}/_{hr}$,” measured “1.3 m above water” (Smith, Derald 14). Along the shore, these winds contributed to a longshore current with a velocity, “of up to 1.3 $^{m}/_{s}$” (Smith, Derald 7).

3.0 Purpose

The purpose of this experiment is to simulate a series of large scale storms on the bathymetry of Lake Athabasca to investigate the possibility of storm-induced large scale sediment transport.

4.0 Theoretical Background

4.1 Steady-State Spectral Wave Model

The foundation for this entire simulation set is the Steady-State Spectral Wave (STWAVE) Model, which can simulate wave conditions across a given bathymetry. The model assumes, “that the relative phases of the spectral components are random” (Smith, Jane 5). As such, The STWAVE model, currently at version 4.0, is phase-averaging, as the data used to get a phase resolving model is typically unavailable (Smith, Jane 5).

Other Important STWAVE Assumptions

- Spatially homogeneous offshore wave conditions
- Steady State Waves, Currents, and Winds
- Linear Refraction and Shoaling
- Depth Uniform Current
- No Bottom Friction
- Linear Radiation Stress

Figure 4.1: Other Important STWAVE Assumptions (Smith, Jane 5)

The other major assumption in the STWAVE model is that there is a, “Mild bottom slope and negligible wave reflection.” The mild bottom slope is self explanatory. In regards to the reflection, both the –x and +x directions (in other words, all reflection) is neglected, as the model is a half-plane model and also does not account for positive reflections (Smith, Jane 4-5). Other important assumptions in STWAVE are summarized in Figure 4.1

The governing equations behind the STWAVE model are given a full summary in *STWAVE: Steady-State Spectral Wave Model - User’s Manual for STWAVE* by referenced throughout this paper. A short overview of the equations, however, will be given.

$$C_r = \frac{w_r}{k} \tag{4.1.1}$$

$$C_{gr} = 0.5C_r\left(1 + \frac{2kd}{\sinh(2kd)}\right) \tag{4.1.2}$$

In this equation, w_r is the angular frequency of the wave, k is the wave number, and d is the water depth.

In the event that currents are present, the wave incidence angle must be changed. First, the wave ray direction μ is computed as a function of C_{gr}, the group celerity, α, the direction of the group celerity, and U and *δ*, which are current parameters and can be neglected in our case.

$$\mu = \tan^{-1}\left(\frac{C_{gr}\sin\alpha + U\sin\delta}{C_{gr}\cos\alpha + U\cos\delta}\right) = \tan^{-1}\left(\frac{C_{gr}\sin\alpha}{C_{gr}\cos\alpha}\right) = \tan^{-1}(\tan\alpha) = \alpha \tag{4.1.3}$$

This reduction yields a much more simplistic STWAVE model. Another equation to govern the wave orthogonal direction under steady conditions is

$$C_{ga}\frac{D\alpha}{DR} = -\frac{C_r k}{\sinh(2kd)}\frac{Dd}{Dn} - \frac{k_i}{k}\frac{DU_i}{Dn} \tag{4.1.4}$$

In this equation, *D is the* derivative, *R* is the, “coordinate in the direction of the wave ray,” and n is the “coordinate normal to the wave orthogonal.” Also, in this case, *k* is the wave number, C_{ga} is the overall wave celerity, and d is the depth. Note that the latter half of the equation, involving the current term U_i can be removed as the simulations in this paper assume no preexisting current (Smith, Jane 7-8).

Also, a conservation equation is used within the STWAVE simulation, one which allows for “steady state conservation of spectral wave action” and is given as:

$$(C_{ga})_i \frac{\partial}{\partial x_i}\frac{C_a C_{ga}\cos(\mu - \alpha)E(\omega_a, \alpha)}{\omega_r} = \sum \frac{S}{\omega_r} \tag{4.1.5}$$

In this equation, C_a is the wave celerity, C_{ga} is the group celerity, the relative celerity, *α,* and *μ*, the absolute celerity, are the same for all simulations in this paper, so the *cos(μ-α)* term reduces to one. *E* is the, “wave energy density divided by (ρ_w × g), where ρ_w is density of water” and g is the acceleration due to gravity.” The value of S is, “the energy source and sink terms” (Smith, Jane 8).

Wind energy can be added to the STWAVE system through the following equation:

$$F_{in} = \lambda\frac{\rho_a}{\rho_w}0.85C_m\frac{u_*^2}{g} \rightarrow \Delta t = \frac{\Delta x}{\mathrm{B}\overline{C}\cos(a_m)} \rightarrow \Delta E = F_{in} \times \Delta t \tag{4.1.6}$$

In the F_{in} equation, λ is a partitioning coefficient (typically 0.75), ρ_a is the density of air, ρ_w is the density of water, C_m is the mean wave celerity, u_* is the friction velocity, and g is the acceleration due to gravity (Smith, Jane 10).

In the Δt equation, Δx is the grid spacing, B is a constant equal to 0.9, C_g as the group celerity, a_m as the wind angle. The combination of the two results in the energy change (Smith, Jane 10).

The STWAVE model also computes wave-wave interactions, white capping, and radiation stress calculation, but the equations are too complex to easily describe here. Please refer to *STWAVE: Steady-State Spectral Wave Model - User's Manual for STWAVE*, pages 11-12 for an in-depth explanation.

The final component of the STWAVE simulation to be discussed is that of breaking waves. These results are highly important to the quality of the results of this report, as the zone of sediment transport is typically assumed to be the width of the zone of breaking. Originally, the wave breaking condition was

$$\frac{H_{mo_{\max}}}{d} = 0.64 \qquad 4.1.7$$

A better way of providing for this equation was made by Miche in 1944 provided a similar equation for, "intermediate to shallow water depths" (Komar 171).

$$\left(\frac{H_\infty}{L_\infty}\right)_{\max} = 0.142 \times \tanh(kh) \text{ (Komar 171)} \qquad 4.1.8$$

which is simplified for STWAVE computation by the following:

$$H_{mo_{\max}} = 0.1 \times L \times \tanh(kd) \text{ (Smith, Jane 9)} \qquad 4.1.9$$

In these cases, H is the wave height, d / h is the depth, k is the wave number, and L is the wavelength.

All the equations discussed thus far are used to output different results from the STWAVE function. The primary outputs are as follows: 1) Wave Height, 2) Wave Frequency, 3) Wave Direction, 4) Radiation Stress, 5) Zone of Breaking. These outputs can then be fed into other analyses. In the scope of this report, two secondary analyses will be used to model the sediment transport based off of the results from the STWAVE outputs. The first is the mass balance approach, while the second is the one-line model approach.

4.2 The Mass Balance Approach

The sediment transport used in the first section of this paper is governed by the Komar formula, which can be stated easily in two parts:

$$I = KP = KECn\sin\alpha\cos\alpha \text{ (Komar 391)} \qquad 4.2.1$$

$$Q = \frac{I}{\rho(s-1)g(1-p)} \text{ (Haas 2)} \qquad 4.2.2$$

These equations can, using input data derived from the STWAVE simulation, provide an output flow rate at all points. The file "break" can be used to roughly determine the zone of sediment transport. The flow rate within that zone can be calculated using the above equations. Then, using the relationship:

$$\frac{dh}{dt} = \frac{dQ}{dy} \qquad 4.2.3$$

where the latter term is in units of (flow rate / distance) / (unit alongshore distance). A uniform "dh" is assumed, and the **gradient** command is used to create $^{dQ}/_{dy}$. Using algebra, the value *d*t can be found, which should theoretically be the storm duration. This approach, however, has a difficult to assume *dh*. Additionally, this method assumes that most of the energy goes into sediment transport, which is not the case.

4.3 The One-Line Model

The second method for analyzing the primary STWAVE results is the one-line model. In this system, an assumption is made that wave and wind motion across an initial shoreline will eventually deform its shape. The system is actually quite simple, consisting only of four primary equations, three of which are run iteratively.

First, the input value of C_{qi} is needed.

$$C_{q_i} = \frac{\rho \sqrt{g/\gamma_b}\, k}{16(\rho_s - p)(1-p)} \bullet H_{b_i}{}^{5/2} \qquad 4.3.1$$

The density constants ρ, ρ_s are the densities of water and sand, respectively, g is the acceleration due to gravity. The value of γ_b is assumed to be 0.5, and the value of k is assumed to be 0.77. A vector containing the breaking depths along the shoreline, H_{bi}, can be computed from the STWAVE output values. Once C_{qi} is computed, the iterative loop can be entered. With an initial shoreline input x_i, modeling of shoreline erosion can begin:

$$\gamma_i^{n+1} = \tan^{-1}\left(\frac{x_i^{\,n} - x_{i-1}^{\,n}}{\Delta y}\right) \qquad 4.3.2$$

$$Q_i^{\,n} = C_{q_i} \sin\left[2\left(\delta_{b_i} - \gamma_i^{\,n}\right)\right] \qquad 4.3.3$$

$$y_i^{n+1} = y_i^{\,n} - \frac{\Delta t}{(h_* + B)\Delta x}\left(Q_{i+1}^{\,n} - Q_i^{\,n}\right) \qquad 4.3.4$$

In this case, *y* is the alongshore distance, and *x* is the shoreline variation difference. Note that this is different from the orientation typically used in one-line models, but is used in this report to provide a uniform axis set throughout the different simulations. Also, Δt can be set to a desired value, but the units of the whole system must agree. Next, the input values from δ_{bi} should be loaded in from the STWAVE data set as the angle of the wave as it begins to break. Finally, the two values, h_* and B, are assumed values, which can be hard to predict. The value of B is the berm height, or the height of sand above the water level, while h_* is the "depth of closure," or the assumed water depth at the edge of the system (Özkan-Haller 18).

5. Experimental Setup and Procedure

5.1 Steady-State Spectral Wave Model

The STWAVE model requires a number of inputs. For the purposes of modeling wave conditions on Lake Athabasca, the required inputs are 1) offshore water depth, 2) wind speed, 3) wind direction, 4) lake bathymetry. Only topographic maps of the Athabasca basin were available. As such, the bathymetry of Lake Athabasca had to first be digitized, and depth samples were approximated at 1 cm intervals at a scale of 1:250,000.

After digitization, a series of limited tests were to be run on the basin to determine the suitability of the digitization. A number of additional simulations were also performed with a bathymetry enhanced by using the **interp2** command in MATLAB. Each simulation was the evaluated and compared to determine which bathymetry type would yield the optimal simulation results.

The selected bathymetry type was then subjected to a number of different wind directions and speeds in an attempt to judge the maximum wave intensity. The conditions which resulted in maximum wave intensity will then have the associated wave breaking results used to determine the zone of breaking around the William River Delta. An assumption was made that sediment transport only occurs within the zone of breaking.

5.2 The Mass Balance Approach

The mass balance sediment transport method, referenced in section 4.2, was then used to calculate the approximate sediment transport duration, under the most likely storm conditions. Three universal input constants are needed in the beginning of the code. The first is g, gravity, which should not change too much from 9.81 $^{m}/_{s}$. Next is the *enhancement_factor*. This value is the decreased scale between data points caused by the use of **interp2** to refine the bathymetry resolution. Lastly, there is the tolerance. It is not unusual for infinite transport rates to be calculated using this method, especially with the assumption of zero sediment transport directly next to a zone of high sediment transport (caused by points being inside and outside the breaking zone). This data causes spikes, which can be remedied by setting a tolerance, in days. Typically, this value should not need to be changed. Other important input values include the Komar constant, k, set to 0.7, *rho*, the density of water, set to 1000. Lastly the specific gravity, s, and porosity, p, of the sediment must be set in the input.

The sediment transport duration is the output from this model, and is calculated by averaging the sediment transport durations over a square 8 x 8 kilometers centered on the tip of the William River Delta.

5.3 The One-Line Model

Similarly, the one-line sediment transport method, referenced in section 4.3, was used to calculate the sediment transport durations. The one-line model requires the definition of several constants. The *enhancement_factor* constant, first mentioned in the previous section (5.2) is needed for the one-line model. Also required is the time step, *dt*, and the number of time steps, *nt*. Other physical constants include, *Rho*, the water density, *Rhos*, the sediment density, p, the porosity, and g, the acceleration due to gravity. The assumed breaking value, *gammab*, is also required and is typically set to 0.5. Finally, the Komar constant, *K*, as well as the constants *hstar, the average water depth, and* B, the above shore sediment berm height, are semi-arbitrary and are selected based on experience. The files *island_dep.dat*, *wavfld*, and *break* are also needed for the one-line model.

Most of these values are highly similar to the mass balance model. The one line method, however, requires more care than the bulk mass balance sediment transport method. As such, extensive smoothing and was applied to the STWAVE input data in order to provide for a more reliable output. Interpolation using the **interp** command was also used to extract more accuracy from the shoreline data.

6. Data Analysis

6.1 Steady-State Spectral Wave Model

Results from the STWAVE simulation provided a view of the wave conditions surrounding the lake. Almost all simulations were run on a storm which produced a steady wind for 50 m/s. This value was chosen as it is the lower threshold for a Category 3 hurricane.

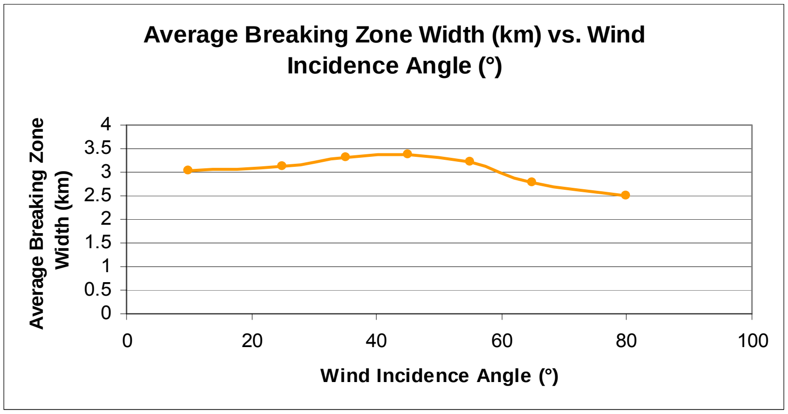


Figure 6.1.1: This plot shows that the maximum zone of breaking is roughly near 45°.

With this assumption in hand, the optimal wind angle was found by comparing the various widths of the zone of wave breaking in a coarse model. The maximum width of the zone of breaking was found near 45°, which is very close to the prevailing southeasterly wind near the William River Delta.

With a wind speed and an incidence direction, the model was ready to run. The offshore boundary depth was simply averaged in input into STWAVE, which was approximately 54 meters. The output of these equations yielded wave heights near the William River Delta were roughly 6 meters high. The waves began to change direction as they approached the delta, with the general trend towards pointing orthogonal to the coastline. The breaking zone was clear and well defined, with an average width of 2.7 kilometers. Note that this value is different from the value seen in figure 6.1.1, as a more coarse resolution was used for that figure, due to the decreased need for accuracy and increased need for general trends.

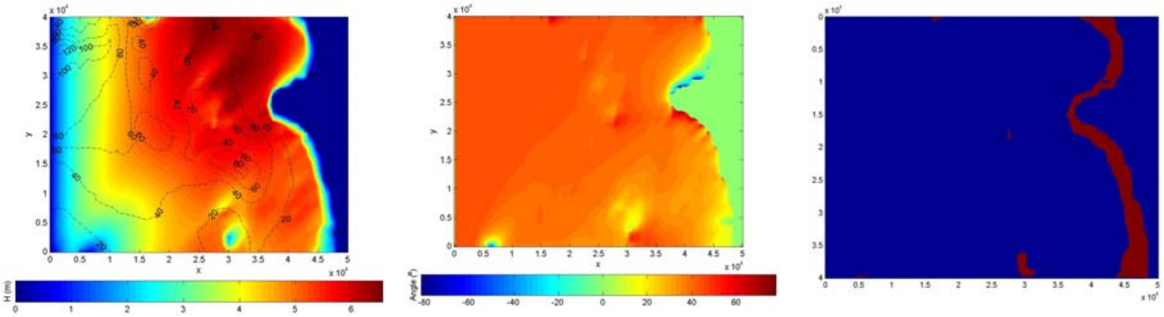


Figure 6.1.2: The varied results from the STWAVE simulation. In order, the resulting graphs are 1) Wave Height Plot, 2) Wave Angle Plot, 3) Wave Breaking Plot

6.2 The Mass Balance Approach

The general observations discussed in the STWAVE section were actually expressed in the large vectors and matrices of data, which were fed into the mass balance sediment transport model. In the simulation itself, a Komar constant of 0.7 was used. Additionally, the water density was assumed to be 1000 $^{kg}/_{m3}$, the specific gravity of the sediment was assumed to be 2.7, and the sediment porosity was assumed to be 0.6. Most importantly, a *dh* of 14 meters was assumed, which is the average water surface depth.

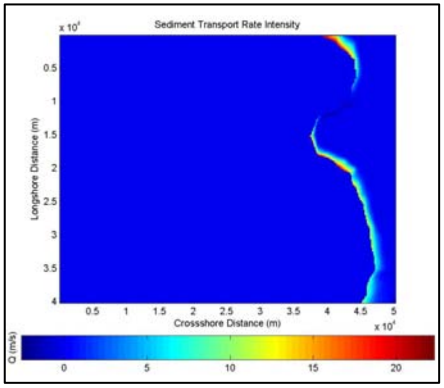
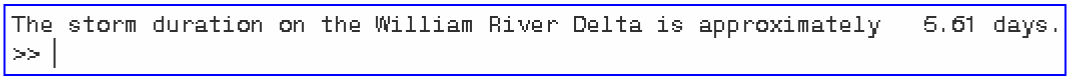


Figure 6.2.1: This plot shows the longshore transport rates along the entire southern Athabascan coast.

These input values yielded a large amount of sediment transport along the coastline. Extremely high levels of sediment transport were found on the extreme eastern end of the simulated area, with moderately high sediment transport rates occurring along the western extent of William River Delta. The computed necessary duration time for sediment transport near the William River Delta was found to be 5.61 days.



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The storm duration on the William River Delta is approximately   5.61 days.
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Figure 6.2.2: The Matlab sediment transport duration output

6.3 The One-Line Model

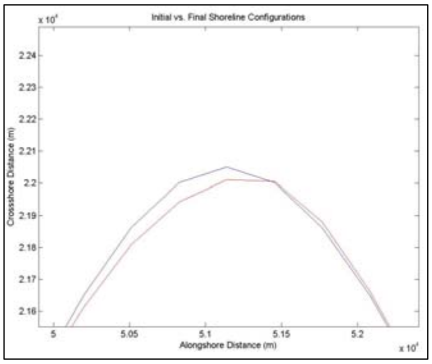
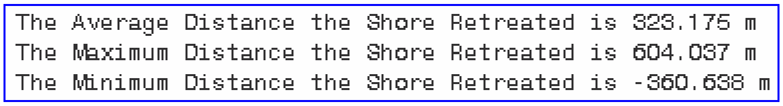


Figure 6.3.1: Shoreline Erosion Using the One-Line Model

The STWAVE data was also run through the one line model simulation. A time step of ten minutes was used for the simulation, with the overall duration lasting six days. The density of water was assumed to be 1000 $^{kg}/_{m3}$ and of sediment to be 2650 $^{kg}/_{m3}$. The porosity of the sediment assumed to be 0.6 and gravity as 9.81 $^{m}/_{s2}$. The guessed values of γ_b, K, h_*, and B, were 0.5, 0.7, 30, and 2, respectively. This simulation showed an average of 323 meters of shoreline degradation. It did not model the actual duration, which was given in a backwards duration as an input. It is thought, from experience, that the end average shore retreat values appropriately approximate the effect of a superstorm event, and thus the simulated run six days can be accepted as the approximate storm duration from this model.



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The Average Distance the Shore Retreated is 323.175 m
The Maximum Distance the Shore Retreated is 604.037 m
The Minimum Distance the Shore Retreated is -360.638 m
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Figure 6.3.2: Matlab Results using the one-line model with the negative retreat being interpreted as shoreline buildup to the east of the William River Delta.

7. Discussion

The sediment transport and duration times show in both models show results which are characteristic with a superstorm in the Lake Athabasca area. The mass balance transport model found an average storm duration across the delta of just less than six days. This number seems to be high, however, it agrees very closely with the results found from the one-line model, the methodologies of which are only related on a basic level. The sediment transport model showed an average retreat of slightly over 300 meters over a period six days. Originally, Derald Smith noted in his report that the superstorm, “planed off the outer 200-400 m of the shelf zone” (Smith, Derald 21). Thus, it can be reasonably assumed that a six day storm in the one-line model corresponds to a storm which has the same effect Dr. Smith describes in his report. The fact that the two simulations independently arrive at roughly the same storm duration, 5.61 and 6 days, seems to indicate the theoretical agreement. These results, however, do not conclude that such storms did exist. Instead, they simply do not exclude the possibility that superstorms have occurred in Lake Athabasca in the past 4,000 years.

8. References

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9. Acknowledgement

I would like to thank Dr. Tuba Özkan-Haller for the immense amount of assistance she provided through the duration of this project. I would also like to thank Dr. Merrick Haller, who also supplied help. Dr. Dan Cox is also owed a great deal of thanks, for administering and orchestrating this REU experience. Next, none of this would have been possible without the work of Alicia Lyman-Holt, whose attention to detail and willingness to make this a wonderful REU experience helped us immeasurably. The Research Experience for Undergraduate (REU) program at the O.H. Hinsdale Wave Research Laboratory was supported by the National Science Foundation (EEC-0244205).

10. Biography

Andrew Collier hails from Wichita, Kansas, and is currently an undergraduate civil engineering major at the University of Notre Dame. His research focus is on computer based applications to civil engineering. Previous to participation in this program, Andrew studied in Perth, Western Australia, where he developed an interest in ocean-based applications of civil engineering. He chose to come to work at the O.H. Hinsdale Wave Lab due to research opportunities at the confluence of his two research interests. In the future, Andrew hopes to attend graduate school in information technology or public policy.