A Summary Study on Arbitrary Lagrangian-Eulerian: Methodology, Implementation, and Application

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Abstract

This summary documentation is part of a report that examined and summarized numerous journal and conference papers and reports focusing on numerical modeling of fluid-structure interaction systems in general, and aeroelasticity in particular. The full report includes an individual summary of each paper and then an appendix attached with the full text of the paper. What is provided here is the overall summary preceding each section. The sections provide general summary and background on the individual papers. The main focus was on fluid-structure interaction and the arbitrary Lagrangian-Eulerian (ALE) reference frame. It is an approach to solving problems in engineering which combines the use of the classical Lagrangian and Eulerian reference frames. Aeroelastic instability is studied in general and the flutter phenomenon is focused on and examined in further detail.

This document is divided up into three distinct sections. The first section will discuss the methodology of the ALE approach. The second section examines the general numerical implementation and applications of the ALE method. Lastly, the third section looks at a specific example of an ALE application in the study of flutter and aeroelastic instability in long span bridges.

Summary and Introduction

Fluid-structure interaction has been an important field of study for many years. Throughout most of its history fluid-structure interaction has been studied through the use of wind tunnel and wave basin testing in addition to other scaled modeling means. While physical modeling can be a very effective and accurate method of research and testing it can be very expensive and time consuming due to numerous tests which must be conducted.

With the drastic increase in computing power over the last decade computer and numerical methods are becoming increasingly more effective. This is leading to increased use and research in using numerical methods to predict fluid-structure interaction motions and other responses. The use of computer modeling to predict fluid and structure motions reduces cost by providing closer estimates of solutions which leads to fewer trial runs when using wind tunnel, wave basin or other modeling facilities. While eventually the computer model may become accurate and consistent enough to no longer require physical modeling, that time may still a ways in the future.

One effective method of computer modeling of fluid-structure interaction systems is the arbitrary Lagrangian-Eulerian (ALE) approach. The current literature on ALE is quite extensive; new literature is coming out constantly. The goal of the first part of this document is to explain and summarize current developments in the ALE approach and to examine the state of the art research being conducted on ALE. This will be done through the analysis of papers on the subject of ALE.

ALE is not just being used in a research setting. There have been several large scale civil engineering projects wherein ALE has provided a substantial benefit through the reduction of the cost and time associated with wind tunnel and wave basin tests and other modeling facilities. The goal of the second portion of this document will be to examine examples of where ALE has been used in structural design and how it affected the design process.

Lastly, this document examines aeroelastic instability and focuses mainly on the study of flutter. Aeroelastic instability is one of the major threats to bridge stability. Flutter specifically has been the cause of several major bridge failures, most notably the failure of the old Tacoma Narrows suspension bridge, in 1940. Since aeroelastic instability is such a threat to bridge stability it is one of the major concerns when designing a bridge, much time and money is spent conducting wind tunnel testing and other modeling. One of the current fields of study is the use of ALE to predict flutter limits in order to limit time and money spent on testing. The third part of this paper looks at aeroelastic instability in general and then looks specifically at flutter in further detail.

In general, this paper will first discuss the ALE approach and look specifically at the methodology. It will secondly focus on the application of the ALE approach and look at its application in engineering design problems. Thirdly, it will look at aeroelastic instability in bridges and focus specifically on flutter in bridges and the use of the ALE approach to solve for the flutter limits.

Arbitrary Lagrangian Eulerian: Methodology

ALE stands for Arbitrary Lagrangian Eulerian. It is an approach to solving problems in engineering which combines the use of the classical Lagrangian and Eulerian reference frames. It is used largely in the analysis of fluid-structure interaction systems. In general, it is beneficial (if not necessary) to use an ALE approach to analyze the structure (and fluid) motions if material strain rate is large and significantly. In other words, ALE is very helpful when analyzing structural motions in which the structure is severely deformed, such as an impact problem or the analysis of a very flexible structure. On their own the Lagrangian reference frame and the Eulerian reference frame can solve most problems.

The Lagrangian Reference Frame is largely used most commonly in solid mechanics. It set up a reference frame by fixing a grid to the material of interest then as the material deforms the grid deforms with it. For example, a beam that has a small amount of deformation typically uses the Lagrangian approach because as the grid deforms it maps out the deflection of the beam due to some load. It also defines the exact displacement of each particle, a factor that can be helpful in tracking motions in solid mechanics. In this method conservation of mass is automatically satisfied because the individual sections of the grid always contain the same (amount of) mass. For structure motions with large deformation in which the grid becomes excessively distorted, the integration time steps become smaller and smaller because they are based on the size of the smallest section of the grid.

The Lagrangian method typically is not the easiest solution for a fluid mechanics problem. The reason is that fluids are not cohesive and so the fluid particles do not stay closely together. So if a grid is mapped out onto a fluid, then no matter how small the initial grid sections the fluid particles will travel independent of each other and diverge in space. This will cause the grid to distort excessively and may overlap each other, and hence will have difficulty providing a reasonable prediction of the fluid motion.

The Eulerian Reference Frame, which is fixed in space, is the typical framework used in the analysis of fluid mechanics problems. It allows for material to flow through the grid. However, it does not track the path of any individual particle. So in motion predictions solved through the Eulerian approach the solution is generally measured in the net flow through a certain area. For the Eulerian method conservation of mass is taken into account explicitly by measuring the flux in and out of each grid section. One of the disadvantages of the Eulerian system is that it does not track the path of any element, in particular the fluid free surfaces.

The arbitrary Lagrangian-Eulerian (ALE) approach combines the use of the two reference frames. It allows for both a flexible grid and a grid that allows for material to flow through it. In essence, it takes the best part of both reference frames and combines them in to one. This is helpful in problems with large deformations in solid mechanics and in fluid-structure interaction. It allows for the grid to track the material to some extent, but when the grid deforms excessively and distorts the aspect ratio of the grid beyond an acceptable point it adjusts the grid and measures the flux of the material during the adjustment of the grid. The difficulty when using the ALE approach is deciding how much to allow a grid to deform and how much flux to allow. This is usually done by setting a limit on the distortion of a segment of a grid and once it deforms past that limit then that part of the grid is remeshed.

When using ALE it requires the input of several variables. These variables allow for the numerical model to compare closely to the real world situation. They allow for the controlling equations to work. There are numerous variables that must be imputed. Several of these include: size, boundary considerations, free surfaces and starting velocities. Intrinsic characteristics of the material are important too. These can include density, viscosity, strength, Young’s modulus, shear modulus and bulk modulus. These variables are necessary to input in order to assure the accuracy of the controlling equations.

The modeling of fluid-structure interaction and of structure-structure interaction requires the use of several controlling equations. There is the conservation of momentum equation. This is usually taken care of with the Navier-Stokes Equations. Conservation of mass is important for when remeshing occurs. When the mesh stays constant, as in an Eulerian approach, there is no need for conservation of mass since it stays constant by default as there is no mass flux. Conservation of energy is also important. Lastly, boundary conditions need to be taken into account. This set of equations controls the motion of the computer model.

When the grid is being mapped it should be done so that each individual element is relatively homogeneous and of the same material. This is important because the computer treats each individual element in the mesh as a constant material through out. If this was not done then it would be difficult to see where the boundary lies between the individual materials.

These equations are then applied to each individual piece of the grid and solved simultaneously. Many meshes may have hundreds of individual grid squares that can lead to vary large matrices for the solution of a problem. This is why large problems will take multiple days of computing time and why very small time steps must be taken. The advances in computing power have dramatically reduced computational time. This has led to more complex problems being solved and also made computer modeling of interaction problems more economical for use in design.

When using ALE to analyze a system it is often times more efficient to only use ALE to model part of the problem and use either Eulerian or Lagrangian to model another part. One example of this would be in a fluid-structure interaction problem. The part of the fluid near the structure would be modeled using ALE and the interface between the two would be modeled using ALE, but the fluid away from the structure may use an Eulerian reference frame because the grid will have no need to flex. It can simply allow for the fluid to pass through it.

When using the ALE approach the grid size is very important. The smaller the individual squares in the grid the more accurate the results; however, the longer it will take to compute. The maximum time step is dependent on the smallest grid size. So when the grid size is really small then it takes more time steps to compute than with a larger grid size. This is why a Lagrangian approach with large deformation leading to very small grid sizes for some squares can be very inefficient. In order to prevent this when there is significant deformation remeshing should take place before the next time step. This will keep all of the grids at least a minimum standard size that will allow for the calculation to be completed.

The ALE approach to analyzing fluid-structure interaction systems is only needed for more complicated problems. As stated earlier, many problems can be solved much simpler using the Lagrangian reference frame or the Eulerian reference frame. However, there are many problems that do not fit either one, and it is necessary to use an ALE reference frame. The ALE approach combines the best parts of the other two reference frames to solve more complicated problems.

Arbitrary Lagrangian Eulerian: Implementation

There are several different types of applications towards which the ALE approach can be applied. They are all situations that would be difficult to analyze in either the Lagrangian reference frame or the Eulerian reference frame individually. Such situations include: free surface, sloshing, and impact modeling, rigid body fluid-structure interaction and flexible body fluid-structure interaction. Each of these is very challenging and important to analyze. The importance of solving these problems through the use of ALE is that most of these problems are normally solved through running tests of models that for some of these experiments can be very expensive. So, if a computer can run a numerical model to get an approximate solution, then it narrows down the number of tests that need to be run.

Analyzing a free surface is done to track the movement of the surface of a fluid. For example, the surface of a lake could be modeled. This application generally only involves a fluid and no structure, but they are difficult to solve using an Eulerian reference frame because the free surface isn’t fixed. If it were done in an Eulerian reference frame there would be elements which would be partially filled. ALE is used to allow for the boundary of the grid to move with the free surface. When using ALE to analyze a free boundary the free surface sets in a boundary condition such that no fluid can flow across the free surface. This maintains that the same elements of the grid remain on the surface and it ensures that the free surface keeps all of the fluid inside it. Free surface situations are generally more efficiently modeled using ALE.

The sloshing application involves the flow of a fluid inside a structure. A typical example of this would be a truck carrying a fluid, like a gasoline truck or a water truck. The fluid inside the structure moves around quite a bit and rotates from side to side. This occurs usually in only partially filled containers since in full containers the fluid will not be able to move as much. The momentum from the sloshing of the fluid inside the container could break the container if it was weak enough or it could cause it to tip over in the case of a truck. ALE is a good way to model this because of the free surface of the liquid in the container and because of the contact with the container.

Impact is important to study because of the wide range of application. Impact is a major concern of every one from automobile makers to the military. Impacts usually have severe distortions that make the ALE approach generally the best approach to solving them. Examples of an impact application would be a bullet hitting a sheet of metal, or a car crash. Impacts are generally considered structure-structure interaction, as opposed to a structure impacting in a fluid that would be fluid structure interaction. Impact problems can typically be modeled well with hydrocodes, which are used with in an ALE system. Hydrocodes have been an important part of solving impact problems. They look at the pressure waves that travel through an object when it is hit and how the wave will reflect through out the structure. These waves are important to look at because the speeds at which they travel and the time it takes for them to reflect back decide how the impact will occur. Currently, automobile makers and the military have the best current numerical programs on impact. Automobile makers use it to limit the number of crash tests they need to perform in order to test safety. Due to small time steps and the degree of accuracy that is required in ALE and since the time steps are so small the pressure waves that travel very rapidly through the structure are now measurable.

The modeling of rigid body fluid-structure interaction is usually a simplification since all structures are flexible to some degree. These applications have a structure that has negligible flexibility and does not affect the fluid flow significantly. These applications can have moving structures the structures just cannot deform significantly. An example would be a piece of metal connected by a hinge immersed in fluid, or a door on a hinge that is being blown around in the wind. The door has motion, but any deformation of the door would be negligible and the deformation would not change the fluid flow. It is all right for the motion of the door to change the fluid flow, which it will, since it will also take place in the model. Something like this would be modeled as a rigid structure with a pin connection in fluid. It could also have a spring connection, a torsional connection or any other kind of connection. This allows for studying the movement of a structure due fluid force, or studying the flow of a fluid around a structure, without worrying about the deformation of the structure that would take more time to compute. An ALE approach would be most useful in this problem as long as the structure is not rigid and rigidly fixed. If this were the case then it would be much simpler to use an Eulerian reference frame. If the structure is moving then an ALE approach is appropriate because the flow of the fluid changes based on the motion of the structure. For something like this a partial ALE approach might save some time since fluid flow which is far away from the structure will not change much, so an Eulerian approach might be best for the flow which does not change. Rigid body problems are good simplifications of more complex problems and can provide a good approximation of what will happen as long as the deformation of the structure is negligible.

Flexible body fluid-structure interaction is one of the most complex and difficult applications currently being studied. Numerical modeling of flexible structures is relatively new and has only been started in the last decade. Flexible bodies can range from bridge decks with a lot of stiffness to a rubber raft with very little stiffness. These situations usually involve the structure deforming at least enough for it to alter the flow of the fluid. Often times the structural distortion can be quite severe on its own. Important examples of flexible body fluid-structure interaction include the oscillation of a bridge deck in the wind, the sway of a skyscraper, or the vibration of a pipe on an offshore oil rig. This topic has been applied to the design of many of the long bridges and tunnels being built. These include the Great Belt East Bridge in Denmark, and a proposed submerged floating tunnel in Norway. When numerically modeling these structures generally one section of a bridge deck or tunnel section is taken at a time and cables and railing usually aren’t included. As of right now it would be much more complicated and time consuming to involve the extra parts of the bridge and only and approximate solution is needed.

These numerical model predictions are generally conducted in combination with wind tunnel and wave basin testing. The goal of the numerical model is to be accurate enough to minimize the number of wind tunnel tests that need to be run. An ALE reference frame must be used for these situations because the complexity of them is too great for either a Lagrangian or Eulerian reference frame. The hardest part of this type of application is the interface between the structure and the fluid. The interesting part about these situations is that the change in flow due to movement of the structure can either dampen or excite the structure. This is discussed next in the section on aeroelastic instability. Fluid-structure interaction with flexible bodies is a very challenging topic and is the newest field with the most room for study in modeling done with the ALE approach.

There are many challenging situations which can be solved using a numerical method based on an ALE approach. These include free surface, sloshing, impact, rigid body fluid-structure interaction and flexible body fluid-structure interaction. An ALE approach is best used to solve situations which do not fit into a Lagrangian or Eulerian setting. The use of a numerical model can provide a close estimate at the correct solution which will minimize the amount of testing which needs to be done.

Arbitrary Lagrangian Eulerian: Application

A case study on potential application of ALE to areoelastic stability is the focus of this section. Instability in bridges caused by wind is the most dangerous threat to bridges. Bridges must be designed to not only support a vertical load, but also support a horizontal load. Also, in the design of the bridge the size and shape must be taken into consideration, along with the strength. The size and shape of a bridge can affect the horizontal load and can also help prevent against various aeroelastic instabilities.

Aeroelastic instabilities can cause twisting and swaying in a bridge deck that can lead to fatigue in the metal and if sever enough it can cause failure of the bridge. The major types of aeroelastic instability include vortex-induced oscillation, galloping, buffeting and flutter. While all of these are important to consider in design this section will focus on flutter after a brief overview of the other three.

Vortex induced oscillation occurs when dealing with bluff bodies such as bridges. The flow around bluff bodies is usually turbulent and vortices are created on the leeward side of the structure due to the wind creating pressure on the windward side and creating suction on the leeward side. These vortices are alternately shed of the back of the structure on the top and bottom of the structure at a rate which depends on the wind speed, length of the body and the Strouhal number. The following equation describes the relationship between these variables.

$$SN = \omega * \frac{B}{V}$$

Where SN is the Strouhal number, ω is frequency of periodic vortex shedding, B is characteristic length of the body, and V is the mean free stream velocity. This vortex shedding creates an oscillatory force acting on the structure at right angles to the direction of the wind. If the frequency of the vortex shedding corresponds with the natural frequency then large amplitudes of vibration can occur. This is called lock in. It can cause long span bridges to noticeably oscillate and it may lead to fatigue failure over time.

In order for vortex induced vibrations to occur several conditions need to be met. The wind direction must be normal to the long axis of the bridge. There needs to be low turbulence. The Reynolds number should be between 30 and 5000. The wind speed needs to stay within 5-12 m/s of the critical region. Lastly, there needs to be very little damping. It needs to be one percent or less of the critical condition. These types of conditions can be satisfied for short to medium span cable-stayed bridges over water and longer span suspension bridges. Vortex shedding can be reduced by making the bridge deck more streamlined. An example of this includes the guide vanes that were put on the Great Belt East suspension bridge in Denmark. These guide vanes successfully suppressed vortex shedding vibration, which occurred at 4 different frequencies and over a wide range of wind speeds. So it is possible to build in devices to prevent vortex-induced oscillations.

Galloping is a form of single degree of freedom areoelastic instability. It is a pure translational, crosswind vibration. This type of instability is typical of slender structures. It is characterized by large amplitudes of oscillation at frequencies much lower than the natural frequency of the structure. It is also more common in square sections. It can be more dangerous to bridge cables since they are much more slender than the bridge deck.

Buffeting is a randomly forced vibration of a structure due to velocity fluctuations in the wind. It is due to turbulence and turbulent eddies from the wake of something up wind. This can cause the bridge itself to vibrate, but is generally more dangerous for trucks and automobiles on the bridge since large obstructions such as a parapet can create wake causing turbulent eddies. These eddies can then for a forced vibration in the car which can cause it to be difficult to steer.

Flutter is a self-induced vibration produced by a change in the wind force as a result of the structures own motion. When wind blows on a bridge the effective angle of attack can generate a vertical force on the bridge which, if it does not act at the center of rotation of the bridge, will create a moment in the bridge causing it to twist. When the bridge twists eventually it will come to a point where the wind is pushing on the other side causing there to be a moment in the other direction. The bridge will oscillate back and forth like this. This causes a certain amount of vibration in the bridge. This vibration is convergent up to a certain point because of the dampening caused by the stiffness of the bridge. However, past a certain speed, known as the flutter speed, the vibration becomes divergent and the stiffness of the bridge is not sufficient to prevent flutter. Flutter is characterized by a rapid build-up in the amplitude of vibration with little or no increase in wind speed. This can cause dangerous instability of the bridge and bridge deck. The divergent nature of flutter is what can cause failure in a bridge.

Flutter was first discovered in airplane wings. When the wings would reach a certain speed they would start to oscillate up and down. In order to prevent flutter the wings were made stiffer, the connection to the plane was made stronger and the wing was made more aerodynamic. Flutter in airfoils occurs at much higher wind speeds than in bridges because airfoils are far more aerodynamic than bridges, which are considered bluff bodies.

There are two types of flutter. They include Stall flutter and Classical flutter. Stall flutter is in the form of single degree of freedom vertical or torsional oscillation. Stall flutter is more common in rectangle or H-sections. Classical flutter is in the form of two degree of freedom coupled vertical and torsional vibration. It is most common in flat plates and airfoils. Classical flutter is the more critical of the two.

All bridge have a flutter speed, but it is the job of the engineer to design the bridge to that the flutter speed is greater than the expected wind speed plus a factor of safety. The critical speed of flutter should be at least 1.3 times the expected speed. Because of the danger inherent in the flutter of a bridge deck determining the flutter speed is a very important part of the design of large bridges. The equations of motion for a two-degree-of-freedom, bluff body can be written:

$$Z'' + 2vZ' + v^{\wedge}2 * Z = \frac{F(t)}{m} + H_1Z' + H_2\theta' + H_3\theta$$

$$\theta'' + 2w\theta' + w^{\wedge}2 * \theta = \frac{M(t)}{I} + A_1Z' + A_2\theta' + A_3\theta$$

Where Z", Z' and Z are the acceleration, the velocity and the initial position of the body. The term θ'', θ' and θ are the angular acceleration, the angular velocity and the initial angular position. The variable v and w are the undamped circular frequencies in still air for vertical motion and rotation, respectively. The term F(t) is the force with respect to time and the term M(t) is the moment with respect to time. The variable m is mass and the variable I is the moment of inertia. The terms H_1, H_2, H_3 and A_1, A_2, A_3 are the flutter derivatives.

The ability of a bridge deck to have flutter instability depends on the magnitude and sign of the flutter derivatives. If H_1 is positive the Galloping will occur. If A_2 is positive then stall flutter will occur. If H_2 and A_1 are positive then classical flutter will occur. In order to determine the values of these flutter derivatives it is necessary to test models either through wind tunnel testing or numerical modeling.

If a bridge is found to flutter at a lower than acceptable wind speed it is necessary to alter the bridge to improve its flutter resistance. There are several possible solutions. The bridge deck can be stiffened which will cause the bridge to dampen the oscillation more which will raise the flutter limit. Another solution is to use a truss instead of a plate girder for the bridge deck. If a truss is used it has holes in it in between each member which allows for the wind to pass through creating a smaller effective surface area which will lower the force of the wind acting on the bridge deck. The truss may also provide more stiffness to the bridge deck. A third solution is to make the bridge more aerodynamic which will increase the flutter limit because more air will go around the bridge instead of into it. Ways of doing this include providing guide vanes or triangular fairings on the ends.

The most famous case of flutter in a bridge section was the collapse of the Tacoma Narrows suspension bridge in 1940. It was the third longest bridge span at the time it was built with a main span of 2800 feet. As soon as the bridge was built it experienced noticeable oscillations, even during periods of relatively low wind speed. The eventual failure occurred due to winds at a speed 35 miles an hour which lasted for 3 hours causing amplitudes of 1.5 feet in the bridge. Then the wind speed increased to 42 miles per hour and created a difference of 28 ft in the elevation of the right side of the bridge from the left side. Higher speed had been recorded but since they were not sustained for as long they did not cause the bridge failure, but they may have caused fatigue in the structural steel.

It had been noticed earlier that the bridge was experience strong oscillations in the wind and so a 1:50 scale model had been made to test in a wind tunnel. Engineers discovered that the bridge was unstable in certain wind and that the oscillations would increase in severity while the wind speed remained constant. This led the engineers to believe that the bridge needed deflector vanes to make it more aerodynamic. A contract for installation of the vanes was under negotiation when the bridge collapsed.

The old Tacoma Narrows Bridge had shallow plate girders for support. While this provide more than enough support for the static loads, it was insufficient to support the torsional loads acting on it. Most other bridges at the time used deep trusses which are stiffer and since they are deeper provide a lot more torsional support. After the collapse, the new Tacoma Narrows Bridge was built and, instead of using plate girders, trusses are used to support the deck.

The old Tacoma Narrows Bridge is an example of why it is necessary to do modeling both physical and numerical before beginning construction. Aeroelastic instabilities can cause severe damage and even failure. With the advances in computer modeling of fluid-structure interaction it should be possible to lower the cost of the necessary physical modeling and ensure that it is done correctly and consistently to prevent such costly failures as the old Tacoma Narrows Bridge.

Flutter and aeroelasticity in general is a complicated example of fluid-structure interaction. It is necessary to conduct numerous physical modeling in order to determine flutter derivatives and then determine the flutter limit of the bridge. Computer modeling is useful to obtain an approximate solution before starting the physical modeling. An ALE approach is the best way to numerically model flutter. It is a perfect example of how a structures deformation, seen in the oscillation of the bridge deck, can change how the fluid acts on it. The most notable example of an ALE approach to numerical modeling being used on a bridge was the Great Belt East Bridge. More than 16 individual box sections were used in the design, each of which needed to be individually tested with a physical model. Each section test in average ran 6 weeks. The use of ALE in numerical modeling to investigate flutter is relatively new and is on the cutting edge of research. The use of the ALE reference frame to successfully numerically model the Great Belt East Bridge happened as recently as December 2003. The use of an ALE approach should be successful in reducing the number of physical tests done and reduce the cost of design.

Concluding Remarks

In conclusion, this paper summarized three main topics. First, it discussed the methodology of the ALE reference frame. Second, it discussed the application of the ALE reference frame. Lastly, it discussed several topics relating to aeroelastic instability and focused on flutter. If there was more time and possibly in the future more extensive investigation will be done in the other aspects of aeroelastic instability and in other applications of the ALE reference frame all together.

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